

Instructions: Complete each of the following exercises for practice.

1. Compute the domain of the vector function $\mathbf{r}(t) = \left\langle \ln(t+1), \frac{t}{\sqrt{9-t^2}}, 2^t \right\rangle$.
2. Compute the following limits.

(a) $\lim_{t \rightarrow 0} \left(e^{-3t} \mathbf{i} + \frac{t^2}{\sin^2(t)} \mathbf{j} + \cos(2t) \mathbf{k} \right)$.

(b) $\lim_{t \rightarrow \infty} \left\langle \frac{1+t^2}{1-t^2}, \arctan(t), \frac{1-e^{-2t}}{t} \right\rangle$
3. Sketch the curve with the given vector function. Indicate the direction in which t increases by an arrow.

(a) $\mathbf{r}(t) = \langle \sin(t), t \rangle$

(b) $\mathbf{r}(t) = \langle \sin(\pi t), t, \cos(\pi t) \rangle$

(c) $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$

(d) $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$

(e) $\mathbf{r}(t) = \langle 3, t, 2-t^2 \rangle$

(f) $\mathbf{r}(t) = t^2\mathbf{i} + t^4\mathbf{j} + t^6\mathbf{k}$
4. Find vector equation and parametric equations for the line segment joining $(2, -3, 1)$ to $(6, 2, -2)$.
5. Do Problems 13.1.21 through 13.1.26 in your textbook (matching some parametric equations to pictures).
6. Show that the curve $x = t \cos(t)$, $y = t \sin(t)$, $z = t$ lies on the cone $z^2 = x^2 + y^2$. Make a sketch of the curve.
7. At what points does the helix $\mathbf{r}(t) = \langle \sin(t), \cos(t), t \rangle$ intersect the sphere $x^2 + y^2 + z^2 = 5$?
8. Find a vector function tracing the curve of intersection of the surfaces given in $\mathbb{R}^3$.

(a) $S_1 : x^2 + y^2 = 4, \quad S_2 : z = xy$

(b) $S_1 : z = \sqrt{x^2 + y^2}, \quad S_2 : z = 1 + y$

(c) $S_1 : z = 4x^2 + y^2, \quad S_2 : y = x^2$

(d) $S_1 : z = x^2 - y^2, \quad S_2 : x^2 + y^2 = 1$

(e) $S_1 : x^2 + y^2 + 4z^2 = 4$ on $y \geq 0, \quad S_2 : x^2 + z^2 = 1$
9. Two particles travel along the curves $\mathbf{r}_1(t)$ and $\mathbf{r}_2(t)$ below. Do they collide? Do their paths intersect?

$\mathbf{r}_1(t) = \langle t, t^2, t^3 \rangle$

$\mathbf{r}_2(t) = \langle 1 + 2t, 1 + 6t, 1 + 14t \rangle$.
10. Suppose $\mathbf{u}(t)$ and $\mathbf{v}(t)$ are vector functions and $f(t)$ is a scalar function. Prove the following.

(a) $\lim_{t \rightarrow a} [\mathbf{u}(t) + \mathbf{v}(t)] = \left[\lim_{t \rightarrow a} \mathbf{u}(t) \right] + \left[\lim_{t \rightarrow a} \mathbf{v}(t) \right]$

(b) $\lim_{t \rightarrow a} [f(t)\mathbf{v}(t)] = \left[\lim_{t \rightarrow a} f(t) \right] \left[\lim_{t \rightarrow a} \mathbf{v}(t) \right]$

(c) $\lim_{t \rightarrow a} [\mathbf{u}(t) \cdot \mathbf{v}(t)] = \left[\lim_{t \rightarrow a} \mathbf{u}(t) \right] \cdot \left[\lim_{t \rightarrow a} \mathbf{v}(t) \right]$

(d) $\lim_{t \rightarrow a} [\mathbf{u}(t) \times \mathbf{v}(t)] = \left[\lim_{t \rightarrow a} \mathbf{u}(t) \right] \times \left[\lim_{t \rightarrow a} \mathbf{v}(t) \right]$